- There are 3.5 hours available for the problems.
- Every problem is worth 10 points.
- Be clear when using a theorem. When you are using an obscure theorem, cite a source.
- Use a different sheet for each exercise.
- If you hand in a draft clearly write DRAFT on the top of that page.
- In this competition we use $\mathbb{N} = \{1, 2, 3, 4, ...\}$. A few other definitions can be found on the back of this document.

MOAWOA

22 mei 2014

Problem 1.

Ten people in a row are numbered 1 through 10. Initially, the person with number i has exactly i coins. Then the following game is played. In every move, a person from 2 to 9 is chosen who has at least two coins. The chosen person then gives simultaneously one coin to his neighbour on the left and one coin to his neighbour on the right. This is repeated as long as possible. The game terminates if each of the persons 2 through 9 has strictly less than 2 coins.

a) Prove that the game always terminates. *(3 points)*

b) Prove that at the end person 4 is the only one without a coin, regardless of the chosen strategy. *(7 points)*

Problem 2.

Let G be a group of which the commutator subgroup $[G, G]$ is a subset of het center $Z(G)$. Suppose that $f : G \to H$ is a homomorphism from G to a group H with the property that the restriction of f to $Z(G)$ is injective. Prove that f is injective.

Problem 3.

Prove that for all $n \in \mathbb{N}$, there are infinite many pairs $(x, y) \in \mathbb{N}^2$ such that

$$x|(n + y^2)$$

$$y|(n + x^2).$$

Problem 4.

Let $\mathcal{F}$ denote the set of strictly increasing functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$\sum_{n=1}^{\infty} \frac{1}{f(n)} = 1.$$

Determine the cardinality of $\mathcal{F}$.

Problem 5.

Let $X_1, \ldots, X_{120}$ be 120 pairwise distinct subsets of the set $Y = 1, \ldots, 2014$. The *MOAWOA-score* of a subset $U \subseteq Y$ is defined as the total number of sets X_i among $X_1, \ldots, X_{120}$ for which the value $\mid X_i \cap U \mid$ is even minus the total number of sets X_i for which this value $\mid X_i \cap U \mid$ is odd.
Prove that there exists a set $U \subseteq Y$ with MOAWOA-score in the interval $[-10, 10]$.

definitions

- **Group**. A group is a set, X, together with an operation $\cdot$ satisfying four properties:
  a) **Closure**: If $a, b \in X$, then $a \cdot b \in X$
  b) **Associativity**: If $a, b, c \in X$, then $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
  c) **Identity element**: There is an $e \in X$ such that $a \cdot e = e \cdot a = a$ for all $a \in X$
  d) **Inverse elemente**: For each $a \in X$ there exists a $b \in X$ such that $a \cdot b = e = b \cdot a$
- **Commutator subgroup** The subgroup that is generated by all commutators $a \cdot b \cdot a^{-1} \cdot b^{-1}$
- **Center** $Z(G) = \{z \in G | \forall g \in G, z \cdot g = g \cdot z\}$
- Group-**homomorphism** A function $f : G \to H$ that takes the multiplication of G to H. I.E. if $a, b \in G$, then $f(a) \cdot f(b) = f(a \cdot b)$.